MACROECONOMIC SOURCES OF THE BOND YIELD “CONUNDRUM”

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Abstract. We estimate a vector autoregression consisting of macroeconomic and financial market variables to uncover the causes of the bond yield “conundrum” of 2004-2005. Using historical decompositions based on the estimated VAR we find that shocks to foreign capital flows, the projected budget surplus, and the risk premium have all contributed to low long-term bond yields during this period. We find that broader measures of capital flows that include private bond purchases have a stronger impact than narrow measures that include only official transactions. This helps explain the discrepancy between RSW (2006), who find no impact of capital flows, and Warnock and Warnock (2005), who find a substantial impact.

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1. Introduction

Historically, long-term interest rates in the United States tend roughly to track movements in the federal funds rate. This relationship can be seen in Figure 1, which shows that large changes in the federal funds rate have been associated with movements in the ten-year bond rate in the same direction: the 372 basis point increase in the federal funds rate from 1987:1-1989:1 was associated with a 211 basis point increase in the ten-year rate; the 300 point increase from 1993:4-1995:1 was associated with a 143 basis point increase in the ten-year rate, and the 185 point increase from 1998:4-2000:2 was associated with a 145 point increase in the ten-year rate.

By contrast, from 2003:4 to 2005:4 the federal funds rate increased by 316 basis points, while the ten-year rate rose a mere 20 basis points. Alan Greenspan (2005) famously characterized this period of rising short-term rates and unchanged long-term rates as a “conundrum.” The behavior of long-term rates is still more puzzling in light of a number of macroeconomic developments that ought arguably to be pushing long-term rates up, including the return of large federal government deficits, the recovery of the economy from the 2001 recession, and the recent increase in inflation.

Bernanke (2006) offers a number of possible explanations for the failure of long-term bond rates to rise in the 2004-05 period. These include a reduction in risk premiums associated with volatility of real interest rates and inflation rates, an increase in foreign official purchases of U.S. Treasury securities, a global savings glut leading to a flow of private capital into U.S. financial markets, and a decline in issues of long-term U.S. Treasury securities.

Recent empirical studies have focused on the impact that risk premiums and capital flows have had on long-term bond rates, with conflicting results. The approach used in several of these
studies is to construct a model of the term structure of interest rates built on no-arbitrage conditions and in some cases a structural macroeconomic model, using the model to decompose movements in interest rates during the “conundrum” period. Kim and Wright (2005), for example, develop and estimate a three-factor term structure model to decompose the ten-year ahead forward rate (derived from the yield on ten-year Treasury bonds) into an expected short rate component and a term premium component. They find that most of the decline in long-term yields from June 2004 to July 2005 can be attributed to a decline in the term (risk) premium, divided about equally between reductions in the premium necessary to compensate for real interest rate risk and the compensation for inflation risk.

Bernanke, Reinhart and Sacks (2004) construct a model that links the factors underlying movements in the yield curve (which are unobservables in the Kim and Wright model) to observable macroeconomic variables – namely the employment gap, inflation, expected inflation, the federal funds rate, and the year-ahead Eurodollar futures rate. Using this “macrofinance” model, Bernanke et al. show that ten-year bond yields were unusually low around 2003-2004 coinciding with a period of heavy foreign central bank intervention in currency markets. They further show that U.S. Treasury yields fell on dates around interventions by the Bank of Japan during this period, suggesting that official capital flows may have been a factor in lowering U.S. Treasury yields in general.

Rudebusch and Wu (2004) go one step further and incorporate a structural New Keynesian macroeconomic model into a no-arbitrage term structure model. Rudebusch, Swanson and Wu (2006) – hereafter RSW – use this model to produce a predicted path for the ten-year Treasury yield during the conundrum period. They then test whether the volatility of interest rates, real GDP growth and inflation, as well as official capital flows, can account for the
deviation from the actual ten-year Treasury yield and the predicted path. In contrast to the studies
discussed above, they find that interest rate volatility accounts for only a small portion of the
unexplained movements in bond yields and other variables had very little effect.

In contrast to the partially structural approaches taken in the papers discussed above,
Warnock and Warnock (2005) use a nonstructural regression analysis to examine the effect on
bond yields of a number of macroeconomic variables including expectations of inflation and real
GDP growth, the budget deficit, the federal funds rate, a measure of interest rate risk, and three
measures of foreign capital flows. They find that all of these variables have explanatory power
for bond yields over the 1984-2005 sample period. In particular, during the conundrum period
foreign official capital flows depressed U.S. bond yields by 100 basis points, while total (private
plus official) capital flows reduced yields by as much as 150 basis points.

This paper builds on previous research regarding the unusual behavior of long-term bond
yields in recent years. As in Warnock and Warnock we examine the influence of a number of
macroeconomic variables – most importantly, we consider broader measures of foreign capital
flows than those examined by RSW. We follow Estrella and Mishkin (1997) and Evans and
Marshall (2001) and model the relationship between macroeconomic factors and bond yields
using a vector autoregression (VAR). Thus our approach, like Warnock and Warnock’s, is
nonstructural. The use of a VAR, however, allows us to account for possible dynamic
relationships between interest rates and the other macroeconomic variables that are ignored in
the simple regression framework. Dynamic interactions would be important if macroeconomic
factors affect bond prices and yields only gradually over time rather than being incorporated
instantaneously due to adaptive learning mechanisms or other departures from an efficient
markets framework. At the same time, dynamic interactions between the macroeconomic
variables may mean that a simple regression mis-allocates responsibility for changes in bond
yields among the independent variables. For example, a regression analysis might attribute a
large fraction of movements in bond yields to changes in the risk premium when in fact the risk
premium is fluctuating primarily because of current and lagged shocks to real GDP growth.
Using a VAR we can identify the ultimate source of fluctuations in bond yields in the form of
underlying structural shocks. We use the estimated VAR to construct a historical decomposition
of bond yields and show which types of shocks have historically (and especially during the
conundrum period) been responsible for movements in bond yields.

Section 2 of the paper describes the data used in our analysis. Section 3 uses our data to
replicate key elements of the Warnock and Warnock and RSW analysis. Since we use quarterly
rather than monthly data (as in the other two papers) and some of our data series differ somewhat
from those used in the other two papers, it is important to test whether we get roughly the same
results with our data as the other authors get with theirs. In addition, it is instructive to test
whether RSW’s finding that capital flows do not account for low bond yields during the
conundrum period is due to their use of a narrow measure of foreign capital flows (official
purchases of U.S. Treasury securities), as opposed to Warnock and Warnock’s broader measures.
Section 4 then describes and reports the results of our vector autoregression, including historical
decompositions. The central finding of our paper is that, in contrast to RSW’s findings, foreign
capital flows do seem to have had a substantial impact on yields during the conundrum period,
though not as large as the effect estimated by Warnock and Warnock. Budget surplus shocks and
shocks to the risk premium also play a large role. Section 5 concludes.
2. Description of data

The aim of this paper is to quantify the impact of macroeconomic variables on long-term interest rates in the United States from 1984-2005 and during the 2004-2005 period (the conundrum period) in particular. We use quarterly data in the empirical analysis. Our proxy for long-term interest rates is the daily average yield on ten-year U.S. Treasury securities for the final month of each quarter. Short-term interest rates are represented by the federal funds rate, also measured as the daily average for the final month of the quarter. The relationship between these variables is shown in Figure 1. A more detailed description of all the data used in this paper is provided in the data appendix.

We use two variables reflecting expectations of future macroeconomic conditions. These are ten-year-ahead expected inflation and one-year-ahead expected GDP growth, both from the Federal Reserve Bank of Philadelphia. Both of these variables have been found to help predict long-term bond yields in a number of studies, including Warnock and Warnock. Figures 2 and 3 show how these variables have behaved over our sample. Expected inflation, shown in Figure 2, trends down from the early 1980s, roughly following the path of actual inflation. Though actual CPI inflation rose during the 2004-05 period, expected inflation has remained constant, indicating that low yields could not have been caused by a decline in expected inflation. Figure 3 shows that expected GDP growth as well as actual GDP growth declined moderately beginning in the first quarter of 2004, which may account for some portion of the decline in long-term interest rates in 2004-05.

To capture the effect of fiscal policy on long-term bond yields we include projections of five-year-ahead budget deficits as a fraction of GDP from the Congressional Budget Office. Engen and Hubbard (2005) and Laubach (2005) find that projected budget deficits are positively
correlated with long-term bond yields. Figure 4 shows the CBO budget forecast and the actual budget surplus as a fraction of GDP. The figure shows that while the deterioration in the fiscal situation from 2001-2003 may have put upward pressure on bond yields, an uptick in both the actual budget surplus and the CBO projected surplus beginning in 2004 could potentially have contributed to lower bond yields during the conundrum period.

An important determinant of the term premium in long-term bond yields is perceptions of risk due to fluctuations in interest rates. Bernanke (2006) argues that reduced interest rate risk – due to reduced volatility of real interest rates as well as inflation – are an important contributor to recent declines in bond yields. In our empirical work we proxy for interest rate risk using a trailing 36-month rolling standard deviation of the change in the ten-year bond yield, shown in Figure 5. This is the same measure used by Warnock and Warnock. Interest rate risk falls in the 1980s to early 1990s, rises briefly in the mid-1990s, and returns to a low level by the late 1990s. There is a slight rise from the late 1990s to 2004 followed by a sharp decline in the conundrum years. This pattern is consistent with Bernanke’s contention that a falling risk premium is behind some of the decline in bond yields in recent years. Other measures of interest rate risk such as those used by RSW also show a reduction in interest rate risk in 2004-05.

Warnock and Warnock and RSW differ most strikingly in their assessment of the impact of capital flows on the ten-year bond yield. RSW look at official purchases of U.S. Treasury securities only. Specifically, their measure of capital flows is the 12-month change in custodial holdings by the Federal Reserve Bank of New York for all foreign official institutions as a fraction of total U.S. Treasury securities in the hands of the public. Warnock and Warnock use the same measure of official purchases of U.S. Treasury securities, normalized by lagged GDP rather than the stock of outstanding debt. Warnock and Warnock consider two broader measures
of capital flows as well: total (official and private) purchases of U.S. Treasury securities, and
total purchases of all types of U.S. bonds (Treasury, agency, and corporate). These data are from
the Treasury International Capital Reporting System (TIC). The data on capital flows from the
TIC system is based on market participants’ reports on purchases and sales of U.S. securities and
is therefore subject to considerable error. Warnock and Warnock adjust the data to correspond
with benchmark surveys of foreigners’ holdings of U.S. securities conducted by the Treasury

In our paper we use capital flow data from the Bureau of Economic Analysis’s
International Accounts which represents the three classes of flows used in Warnock and
Warnock. As in Warnock and Warnock, the BEA constructs its data using data from the TIC
system. The BEA has a different system for adjusting its flow data in line with the surveys. We
compare our results using the BEA measure of official capital flows with those using the FRB
New York data. In all cases we deflate the capital flow figures by current-period nominal GDP.

Figure 6 shows the three BEA measures of capital flows. For expository purposes we
show the four-quarter cumulative flows, though in most of the empirical work we use single-
quarter flows. Each of the capital flow measures shows rising capital inflows into the late 1990s
followed by a dramatic decline around the middle of 1997. Official purchases and total purchases
of U.S. Treasuries stayed low or negative through 2000, turning up strongly in 2001. Total net
foreign inflows into the U.S. bond market also fell after 1997, but recovered strongly beginning
in 1998. Official purchases of U.S. Treasuries rose sharply in 2003-2004 and then fell just as
sharply in 2005. With private net Treasury purchases roughly constant during this period, total
net purchases of U.S. Treasuries followed the same pattern. By contrast, total purchases of all
U.S. bonds rose steadily until the end of 2004 and then leveled off rather than falling. These
differences suggest that estimates of the impact of capital flows will differ substantially depending on whether one uses a broad or narrow measure.

Figures 7 and 8 compare the BEA data to some of the data series used in other studies. Figure 7 compares the BEA data on total purchases of all U.S. bonds with the corresponding raw data from the TIC that is the basis of Warnock and Warnock’s broadest measure. The data follow each other very closely, with the raw TIC data showing higher levels of capital inflows on average after 1996. As shown in Warnock and Warnock’s Figure 3, the TIC data tend to overstate capital inflows during this period relative to the benchmark surveys. Warnock and Warnock’s correction produces a series very similar to the BEA data (see their Figure 1). Figure 8 also shows a close correspondence between the BEA and FRB New York measures of official net purchases of U.S. Treasuries. Thus our data is comparable to that used in previous studies.

3. **Extensions of earlier studies**

As a preliminary step in our empirical analysis we replicate some of the work in the Warnock and Warnock and RSW papers using our data. This exercise serves three purposes. First, we are able further to verify that our data conforms in large part to the data used in these studies. Second, we can show the extent to which the key distinction between the results in these two papers regarding the effect of capital flows depends on which measure of capital flows is used. Third, the detailed discussion of the approaches taken by those authors helps motivate our use of the VAR methodology.

The centerpiece of Warnock and Warnock’s analysis is a single-equation least squares regression of the ten-year bond yield on the federal funds rate, expected inflation, expected GDP growth, a measure of the risk premium, the lagged budget deficit, and capital flows. The
regression also includes the difference between the one-year-ahead and ten-year ahead inflation forecasts; we use data from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia for both of these variables. Warnock and Warnock impose the restriction that the coefficients on the federal funds rate and expected inflation sum to one, which is tantamount to assuming that the real interest rate is stationary. Our regression departs from Warnock and Warnock’s in that we use the CBO’s five-year budget projection in place of the lagged budget deficit and we use BEA capital flows data in place of the benchmark-adjusted TIC data used in Warnock and Warnock. We also use quarterly rather than monthly data.

The results of this regression using our data are reported in Table 1. For the most part the estimates are quite similar to those in Warnock and Warnock. Coefficient estimates on the federal funds rate and ten-year inflation expectations are virtually identical. Unlike Warnock and Warnock, the coefficients on the difference between one-year and ten-year-ahead inflation expectations and expected GDP growth are close to zero and insignificant. Interest rate risk is positively correlated with ten-year bond yields – though the coefficient is smaller than in Warnock and Warnock – and is statistically significant. The budget surplus forecast is also statistically significant and has an effect of about the same magnitude as in Warnock and Warnock. Finally, all of the capital flows variables are statistically significant at at least the ten percent level. A one percentage point of GDP increase in capital flows (in 2006, approximately $130 billion) results in about a third of a percentage point decrease in the ten-year bond yield regardless of the measure used. In Warnock and Warnock official flows have a stronger effect (a one percentage point of GDP increase is associated with about a 43 basis point reduction in the ten-year bond yield) than total purchases of all types of bonds (about a 23 basis point reduction).
RSW construct a dynamic macrofinance model of the term structure of interest rates to explain low bond yields during 2004-2005. The macroeconomic component of the model is a four-equation New Keynesian model in which inflation ($\pi_t$), the output gap ($y_t$), and two latent variables representing components of monetary policy – the medium-run inflation target ($L_t$) and a cyclical component ($S_t$) – are determined jointly:

$$
\pi_t = \mu_\pi L_t + (1-\mu_\pi)\left[\alpha_{\pi 1}\pi_{t-1} + \alpha_{\pi 2}\pi_{t-2}\right] + \alpha_y y_{t-1} + \varepsilon_{\pi t} \tag{1}
$$

$$
y_t = \mu_y E_{t+1} y_{t+1} + (1-\mu_y)\left[\beta_{y 1} y_{t-1} + \beta_{y 2} y_{t-2}\right] - \beta_r (i_{t-1} - L_{t-1}) + \varepsilon_{yt} \tag{2}
$$

$$
L_t = \rho_L L_{t-1} + (1-\rho_L)\pi_t + \varepsilon_{L_t} \tag{3}
$$

$$
S_t = \rho_S S_{t-1} + (1-\rho_S)\left[\gamma_y y_{t} + \gamma_\pi (\pi_t - L_t)\right] + u_{St} \quad , \quad u_{St} = \rho_u u_{St-1} + \varepsilon_{St} \tag{4}
$$

where $i_t$ is the interest rate on one-month Treasury bills (which tracks the federal funds rate closely and so can be used to proxy for the monetary policy instrument) and the $\varepsilon_{xt}$’s are disturbance terms. The finance component of the model derives yields on bonds of different maturities from information on the one-month rate, the macroeconomic variables, and the two latent factors. The model imposes no-arbitrage conditions and assumes that the market price of risk is an affine function of the state variables of the economy, with the result that the yield on an n-month bond at time t ($i_t^n$) is given by

$$
i_t^n = (1/n)[a_n + b_n X_t] \tag{5}
$$

where $X_t$ is the vector of state variables (in this case current and lagged values of inflation and output, current values of the two latent variables, and the disturbance term $u_{St}$) and $a_n$ and $b_n$ are
vectors that are defined recursively (see RSW and Rudebusch and Wu (2004)). RSW estimate
this model for the period January 1988 to December 2000. They then use the estimated model to
make projections of the (zero coupon) ten-year bond rate for the period January 2001 to
December 2005. They find that their model tracks the actual yield data well up to 2004, but that
from the second half of 2004 through 2005 the actual ten-year bond yield is considerably below
the level predicted by the model. The mean difference over this period is 33 basis points – this is
a measure of the bond yield “conundrum”.

RSW then seek to determine whether measures of interest rate volatility and capital flows
can account for the disparity between the observed ten-year bond yields and those predicted by
the model. They do this by running a regression of the model residuals (actual minus predicted
values) on three measures of interest rate volatility, two macroeconomic volatility measures
(realized volatility of GDP growth and core PCE inflation), and the FRB New York measure of
foreign official purchases of U.S. Treasury bonds. They find (see their Table 7) that while most
of the volatility variables are correlated with the model residuals, the coefficient on official
capital flows is small and statistically insignificant. Given the change in official capital flows
over the 2004-2005 period, they find that capital flows cannot account for the interest rate
conundrum. In fact (see their Table 8), the combined effect of all the variables in their regression
amounts to less than 30 percent of the prediction error in the 2004-2005 period, leaving over 70
percent of the conundrum unexplained.

We replicate RSW’s analysis using data supplied by Eric Swanson. Specifically, we run
regressions with the model prediction errors from the RSW model as the dependent variable and
the variables in Table 1 as independent variables. The model residuals are converted to quarterly
frequency using last month of quarter values. Results for these regressions are shown in Table 2.
As in RSW’s analysis, the correlation between the model residuals and the explanatory variables – especially the capital flow variables – is weak, evidence of the ability of the model to pick up most macroeconomic influences on bond yields. Of particular interest, however, are the results in the last column, corresponding to regressions in which total purchases of all types of U.S. bonds is used as the measure of capital flows. In all regressions the BEA measures of capital flows enter with the expected negative sign, but in the final regression the coefficient is considerably larger and statistically significant. The results suggest that a one percentage point of GDP increase in total foreign flows into the U.S. bond market is associated with a 9.73 basis point drop in the ten-year bond yield relative to the level predicted by the RSW model. In the same regression the budget projection enters significantly with the expected sign as well – a one percentage point of GDP increase in the five-year out budget projection is associated with a 4.51 basis point drop in bond yields relative to that predicted by the model. Expected inflation enters with the wrong sign, possibly because the model already accounts for movements in expected inflation through the “level” latent variable $L_t$.

Table 3 reports results of decompositions of the bond yield conundrum similar to those performed by RSW. The first row of the table shows that the residual from the RSW model fell by 52.2 basis points from 2004:1 to 2005:4; that is, the actual bond yield was 52.2 basis points below the level predicted by the model. The other rows report the estimated impact of each variable based on the Table 2 regressions, computed by multiplying the coefficient estimate from Table 2 by the change in each variable from 2004:1 to 2005:4. Figures in the first three columns show that none of the variables has a meaningful impact on the model residuals. In the case of BEA estimates of official and total purchases of U.S. Treasury securities, the reason is that these flows declined during this time period. In the case of the FRB New York measure of official
purchases, the negative impact is due to the “incorrect” sign on capital flows in Table 2 combined with a drop in purchases over the period. The increase in the projected budget surplus is found to have a very small impact in the expected direction.

The figures in the final column, corresponding to the regressions in which total purchases of all U.S. bonds is used as the measure of capital flows, show that increased capital flows, increased budget surplus projections, a decline in interest rate risk, and a drop in expected GDP growth all had a small negative impact on interest rates. But the combined effect is only -18.4 basis points, or just over one third of the total drop in rates unexplained by the RSW model. This result is consistent with RSW’s decompositions, though our results indicate a larger role for capital flows (broadly measured) than they find. However, when we define the conundrum period to be 2004:2-2005:2 to correspond to RSW’s time frame (June 2004 to June 2005), we find no substantial impact for any variable, including broadly-defined capital flows (results not shown). This is due to the fact that over this narrower period most of the variables, including broad capital flows, moved in a direction that would tend to push interest rates up rather than down.

The question asked in the decompositions reported in Table 3 is, to what extent can the decline in ten-year bond yields from 2004:1 to 2005:4 (relative to the predictions of the RSW model) be explained by changes in macroeconomic variables over the same time period? An alternative and arguably just as meaningful formulation of this question is, to what extent can the low average bond yields over this period (relative to the levels predicted by the model) relative to their average values in a previous period be explained by differences in the average value of the macroeconomic variables relative to the same earlier period? Decompositions to address this question are reported in Table 4. The first row of the table reports the average of the residuals
from the RSW model from 2004:1-2005:4 minus their average values from 1990:1-2003:4. During the conundrum period the residuals averaged 33.7 basis points below their average level during the earlier period; since the model tracks actual bond yields accurately during 1990:1-2003:4 (the average residual during this period is only about four basis points), most of the difference is due to the fact that actual bond yields were lower than the model’s predictions during the later period. The size of the conundrum measured in this way is very close to that in RSW (32.2 basis points). The first three columns of Table 3 present a message similar to that in Table 2 – none of the variables accounts for much of the bond yield conundrum, though official and total purchases of U.S. Treasuries using the BEA measure are shown to have a modest negative impact on bond yields. But the final column of Table 3 tells a strikingly different story: capital flows broadly defined can account for essentially the entire interest rate conundrum.

The exercises in this section lead us to some preliminary conclusions. First, the macroeconomic variables we have identified do have explanatory power for ten-year bond rates. Second, we can use our data to replicate roughly the results in the Warnock and Warnock and RSW papers. Third, the estimated impact of capital flows on ten-year bond yields seems to depend crucially on whether capital flows are measured narrowly (official purchases of Treasury securities) as in RSW or broadly (official and private purchases of all U.S. bonds) as in Warnock and Warnock.

As RSW state, care must be taken in interpreting the results of the regressions and decompositions based on their model residuals. The fact that a variable is correlated with the residuals from their model does not mean that it would continue to have an impact if it was incorporated in the model to begin with. In addition, neither the Warnock and Warnock nor the RSW approaches incorporate dynamic interactions between bond yields and the macroeconomic
variables that are candidates for explaining the bond yield conundrum (of course, the RSW model does incorporate dynamic relationships between bond yields and the macroeconomic variables that are built into the model). Furthermore, both approaches suffer from the potential endogeneity of the macroeconomic variables. Capital flows, for example, might respond contemporaneously or with a lag to variables like expected inflation and GDP growth, in which case the estimated effect of capital flows may represent the combined effect of capital flow and other “shocks.” The VAR analysis in the next section resolves these problems: all of the variables are incorporated in the model determining bond yields from the outset, dynamic interactions are accounted for, and structural shocks are identified. These advantages come at a cost, of course. In particular, we sacrifice the efficiency gains inherent in a structural approach such as that in RSW. We regard our VAR approach as a useful supplement to the papers described above.

4. Vector autoregression analysis

We estimate a VAR of the form

\[ A(L)X_t = B\varepsilon_t \]  

where \( X_t = (i_{10,t}, \pi_{t+10}, f_{1}, \pi_{t+10}, \text{Capital flows}_t, \ y_{t+1}, \text{Risk}_t, \text{Sur}^{\varepsilon}_{t+5}) \), \( \varepsilon_t \) is a vector of corresponding structural shocks with diagonal variance-covariance matrix \( \Omega \), \( A(L) \) is a fourth-order polynomial in the lag operator, and \( B \) is a lower diagonal matrix. Variables are defined as in Table 1. We use the real ten-year bond and federal funds rate in order to reduce the dimensionality of the system to a more manageable size. In doing so we impose the constraint that movements in inflation
expectations are transmitted within the quarter to the bond rate and the Federal Reserve’s target federal funds rate. With this transformation all the variables in the VAR are arguably stationary.

This recursive model structure imposes the assumption that within the quarter each variable in $X_t$ affects only the variables ahead of it in the ordering and is affected only by the variables after it in the ordering. For the last three variables the choice of ordering is innocuous – it is reasonable to assume that their corresponding shocks are orthogonal, and in fact the residual covariance matrix from the estimated VAR confirms that this is the case. The ordering of the other three variables is more problematic on theoretical grounds and based on the estimated covariance matrix. Since we use last-month values for the two interest rates and total-for-the-month capital flows we can argue that capital flows affect the interest rate series rather than the other way around. The assumption that the federal funds rate affects the ten-year bond yield within the quarter but not the other way around reflects the infrequency of FOMC meetings and the tendency for the Fed to respond gradually to economic events like changes in bond yields. With these assumptions we can identify the structural shocks in $\varepsilon_t$ using the Cholesky decomposition of the residual covariance matrix.

Figures 10 to 13 show the estimated response of the real ten-year bond yield to the six structural shocks in four different specifications of the model corresponding to different measures of capital flows. In all cases the impulse response functions seem reasonable. Positive shocks to the real bond rate, the real federal funds rate, expected GDP growth and interest rate risk cause the real bond rate to rise, while positive shocks to capital flows and the budget surplus cause the real bond rate to fall. For all variables but the projected budget surplus the short-term effects are statistically significant. Each of the capital flow variables has a similar effect on the
real bond rate. While the effect of official purchases is short-lived, the broader measures have slightly stronger and considerably more persistent effects.

The estimated VAR can be used to construct a historical decomposition which shows how the observed history of structural shocks combined to generate movements in each of the endogenous variables. We construct historical decompositions of the real ten-year bond yield using a simulation method. We first use the residuals from the estimated VAR and the estimate of B in equation (6) to recover series for each of the structural shocks. We derive a “baseline” prediction for the real ten year bond yield by setting all the shocks to zero and then, using 1984 values of the endogenous variables as initial values, using the model to forecast the bond yield from 1985:1 to 2005:4. To compute the movements in the bond yield attributable to shocks to a particular variable we generate a new forecast using the realized values of the shocks to that variable and setting all other shocks and initial values of the endogenous variables to zero. By construction (given the linear nature of the model), the baseline forecast and the forecasts corresponding to each shock sum to the actual real bond rate series.

Figure 14 shows the actual and baseline forecast of the real ten-year bond yield for four VARs corresponding to alternative measures of capital flows. The figures show that in three cases the effect of initial conditions disappears by the conundrum period, with actual levels of the bond yield considerably below the baseline forecasts from 2004:1 to 2005:4. This implies that shocks occurring after 1985:1 had a net negative impact on bond yields during this period. In the VAR using the broadest measure of capital flows initial conditions still played a role in the 2004:1 to 2005:4 period, resulting in a smaller discrepancy between the actual bond yield and the baseline forecast.
The results of the historical decompositions for the conundrum period 2004:1 to 2005:4 are summarized in Table 5 and Figure 15. The top row of Table 5 shows the total discrepancy to be explained: real long-term bond rates averaged from 58.1 to 125.2 basis points below the baseline projection from the estimated VARs. In most cases the largest contributor to low bond rates was shocks to the real federal funds rate, accounting for between 27.0 and 48.1 basis points of the difference, or between 24.6 and 49.1 percent of the difference.

The next largest contributor is shocks to the budget surplus, which accounts for between 33.1 and 38.1 basis points (27.6 to 57.1 percent) of the decline in the ten-year bond yield. The general trend since 2001 towards large budget deficits would suggest upward rather than downward pressure on interest rates. This result therefore bears some explanation. As shown in Figures 10 to 13, shocks to the budget surplus are estimated to have their maximum impact on the ten-year bond rate several quarters after the shock. Taking the model using total purchases of U.S. Treasuries – the impulse responses from which are displayed in Figure 12 – as an example, it seems that the large impact on the ten-year bond yield in early 2004 would have been the result of shocks to the surplus projection occurring between 2000 and 2003. This supposition is borne out by Figure 16, which shows the actual path of the projected budget surplus from 2000:1 to 2005:4 as well as the estimated structural shocks from this version of the VAR. Except for two large downward innovations in the projected surplus in 2001:2 and 2003:2 (corresponding to passage of major tax cuts), most of the innovations in 2000 to 2003 were positive and were offset only partially by small negative innovations in late 2003 and early 2004. As shown in Figure 15, the negative impact of budget surplus shocks was felt most strongly in 2004; by the end of 2005 budget surplus shocks had a much smaller negative impact on the ten-year bond yield.
In suggesting that shocks to the projected budget surplus in 2000-2003 exerted downward pressure on the ten-year bond yield in 2004-2005 we are not saying that bond markets incorporate relevant information only with a lag of several years. Rather, we are saying that the shocks to the budget surplus that occurred in 2000-2003 had effects on other macroeconomic variables that reverberated through the economy over time, culminating in lower interest rates several years afterwards. A caveat to these results, however, is that the response of the ten-year bond yield to projected budget surplus shocks is measured with considerable error, so the true impact may be much smaller (or larger) than that shown in Table 5.

The effect of capital flows on the ten-year bond rate is also considerable. Official flows are estimated to have reduced the bond yield by an average of 11.1 or 19.1 basis points, depending on whether the BEA or FRB New York measure is used. When the broadest measure of capital flows is used shocks to capital flows account for a 15.0 basis point reduction in interest rates, over 25 percent of the total size of the conundrum. When private and official flows of Treasury securities is considered, the estimated effect is to decrease bond yields by an average of 37.4 basis points over the period, amounting to over a third of the discrepancy between actual and baseline yields. These results are roughly consistent with the results in section 3 – the estimated effect of capital flows tends to be larger the broader the measure used (though in section 3 it was total purchases of all types of bonds that produced the strongest effect). In general capital flows had a larger impact on bond yields early in the conundrum period. In fact, negative shocks to FRB New York official flows in late 2004 and early 2005 actually put upward pressure on bond rates during that period (as shown in Figures 10-13, these shocks would have had an immediate impact on bond yields). But by 2005:4, capital flows are estimated to account for a large decline in bond yields, ranging from 20 to 40 basis points.
The risk premium also had a substantial effect on bond rates on average in three of the four models, accounting for a 16.3 to 17.2 basis point reduction in the ten year bond yield. The exception is the model using the broadest measure of capital flows, which shows that the average effects was to push bond yields up modestly. The average figures, however, disguise substantial differences in the impact of interest rate risk in the early and later parts of the conundrum period. As shown in Figure 15, the net effect of interest rate risk in 2004 was positive, while by the end of 2005 the effect was strongly negative. In fact, by 2005:4 the models estimate that interest rate risk reduced bond yields by between 40 and 60 basis points, depending on the way capital flows are measured. Thus Bernanke’s contention that a decline in the risk premium is an important explanation for the conundrum is supported for the later period but not the earlier period.

Finally, shocks to expected GDP growth are estimated to have had a small positive impact on bond rates during the conundrum period, particularly in early 2004 and late 2005. A final component of the discrepancy between bond yields and the baseline projection – ranging from 1.3 basis points to 20.6 basis points – is due to shocks to the ten-year bond yield itself, which can be interpreted as the fraction of the bond yield conundrum that is not explained by the variables in the model.

5. **Interpretation of results and concluding comments**

The results of the vector autoregression exercise in the previous section suggest that the unusually low yield on ten year bonds from 2004 to 2005 – a period in which short-term interest rates were rising sharply – can be explained by a number of factors. Foreign capital flows, a declining risk premium, and the lagged effects of favorable budget surplus shocks all contribute to some extent to explaining the conundrum. Capital flows are estimated to have a larger effect
when measured broadly. Declining interest rate risk has had its greatest impact in recent quarters: in 2005:4, low interest rate risk is estimated to be the largest single factor explaining the low bond yields. Positive budget surplus shocks in the early 2000’s played a role in generating low bond yields throughout the period, more so in 2004 than in 2005. The models predict that adverse budget surplus shocks beginning in 2003 should put upward pressure on bond yields in quarters and years to come.

The results regarding capital flows are especially noteworthy. Warnock and Warnock (2005) found that capital flows had a large impact on bond yields during 2004-2005. We find a smaller but still substantial effect. RSW found that capital flows did not have an impact on bond yields. Our VAR analysis and our replication of RSW’s regressions show that their finding is dependent on use of a narrow measure of capital flows. We find that official purchases of U.S. Treasury securities have a relatively modest impact on bond yields (especially during late 2004 and early 2005), but measures that include private purchases of Treasuries and purchases of other U.S. bonds have a much larger effect. This finding may be due to the close substitutability between official and private purchases and between purchases of Treasuries and other bonds. For example, from 2004:1 to 2005:4, total foreign purchases of all types of U.S. bonds increased from 5.49 percent of GDP to 6.19 percent of GDP. We would expect this inflow of capital to put downward pressure on U.S. bond rates in general, including yields on U.S. Treasury securities – except in the unusual circumstance of a general flight out of Treasuries towards other securities. It would also put downward pressure on exchange rates outside the U.S., relieving the pressure on countries with explicit or implicit dollar pegs to undertake official purchases of U.S. securities. Hence official purchases fell during this period from 2.37 percent of U.S. GDP to 0.67 percent. This period of falling official purchases and falling bond yields within a longer sample period
would contribute to a weak overall correlation between official purchases and bond yields, whereas overall purchases of U.S. securities would continue to show a negative correlation to U.S. bond yields.

This interpretation of the relationship between different measures of capital flows and the empirical results reported above suggest that it is not official capital flows per se that have contributed to low U.S. bond yields in recent years, but private capital flows supplemented from time to time by official flows. That is, the evidence is more consistent with Bernanke’s “savings glut” hypothesis than the idea that bond yields are low because of currency manipulation by foreign central banks.
Data Appendix

Deficit projections. CBO five-years-out projection, as a percent of nominal GDP. Data provided by Thomas Laubach.


Federal funds rate. Federal Reserve Bank of St. Louis.


Risk premium. Trailing 36-month rolling standard deviation of change in ten-year bond rates.

Foreign capital flows:

Official Treasuries purchases (FRB New York). Federal Reserve Bank of New York. Change in position of FRB holdings of Treasury securities on behalf of foreign official agencies to last month of quarter from corresponding month one year before. As percent of GDP.

Official Treasuries purchases (BEA). Bureau of Economic Analysis. U.S. International Transactions, Table 1 line 58. As percent of GDP.

Official and private purchases of U.S. Treasuries. Bureau of Economic Analysis. U.S. International Transactions, Table 1 line 65. As percent of GDP.

Total purchases of all U.S. Bonds. Bureau of Economic Analysis. U.S. International Transactions: for 1984-1997, Table 1 line 57 plus Table 1 line 65 plus Table B line B10; for 1998-2005, Table 1 line 57 plus Table 1 line 65 plus Table A line B16 plus Table A line B30. As percent of GDP.

References


Greenspan, Alan (2005), Testimony Before the Committee on Banking, Housing, and Urban Affairs, United States Senate, February 16.


<table>
<thead>
<tr>
<th></th>
<th>Official-FRBNY</th>
<th>Official-BEA</th>
<th>Official + private Treasuries</th>
<th>All bonds</th>
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<tr>
<td>$\pi_{t+10}$</td>
<td>0.66</td>
<td>0.67</td>
<td>0.72</td>
<td>0.76</td>
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<td></td>
<td>(0.11)</td>
<td>(0.10)</td>
<td>(0.07)</td>
<td>(0.08)</td>
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<tr>
<td>$f_t$</td>
<td>0.34</td>
<td>0.33</td>
<td>0.28</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.10)</td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
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<td>$\pi_{t+1} - \pi_{t+10}$</td>
<td>-0.23</td>
<td>-0.12</td>
<td>-0.03</td>
<td>0.29</td>
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<td></td>
<td>(0.48)</td>
<td>(0.44)</td>
<td>(0.36)</td>
<td>(0.31)</td>
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<tr>
<td>$y_{t+1}$</td>
<td>-0.10</td>
<td>-0.01</td>
<td>-0.06</td>
<td>0.20</td>
</tr>
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<td></td>
<td>(0.20)</td>
<td>(0.19)</td>
<td>(0.15)</td>
<td>(0.18)</td>
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<tr>
<td>Risk$_t$</td>
<td>2.46</td>
<td>2.48</td>
<td>2.59</td>
<td>2.66</td>
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<td></td>
<td>(0.79)</td>
<td>(0.80)</td>
<td>(0.68)</td>
<td>(0.65)</td>
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<tr>
<td>$Sur_{t,5}$</td>
<td>-0.19</td>
<td>-0.18</td>
<td>-0.21</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.04)</td>
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<tr>
<td>Capital flows$_t$</td>
<td>-0.34</td>
<td>-0.37</td>
<td>-0.37</td>
<td>-0.32</td>
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<td></td>
<td>(0.19)</td>
<td>(0.14)</td>
<td>(0.10)</td>
<td>(0.07)</td>
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<tr>
<td>R-squared</td>
<td>0.78</td>
<td>0.78</td>
<td>0.82</td>
<td>0.84</td>
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Newey-West standard errors in parentheses. Bold = statistically significant at 5% level, Italics = statistically significant at 10% level. Regression equation:

$$i_{10,t} = \alpha_0 + \alpha_1 \pi_{t+10}^e + (1-\alpha_1)f_t + \alpha_2 y_{t+1}^e + \alpha_3 \text{Risk}_t + \alpha_4 Sur_{t,5}^e + \alpha_5 \text{Capital flows}_t + \varepsilon_t$$

where

- $i_{10,t} =$ ten-year bond yield (daily average, last month of quarter).
- $f_t =$ federal funds rate (daily average, last month of quarter)
- $\pi_{t+1}^e =$ one-year ahead CPI inflation forecast from the SPF
- $\pi_{t+10}^e =$ one-year ahead CPI inflation forecast from the SPF
- $y_{t+1}^e =$ one-year ahead GDP growth forecast from the SPF
- Risk$_t =$ trailing 36-month rolling standard deviation of change in ten-year bond yield
- $Sur_{t,5}^e =$ CBO 5-year projection of budget surplus as a percent of GDP
- Capital flows$_t =$ capital flow measures described in text and at top of columns

<table>
<thead>
<tr>
<th></th>
<th>Official-FRBNY</th>
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<th>Official + private Treasuries</th>
<th>All bonds</th>
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<tr>
<td>$\pi_{t+10}'$</td>
<td>5.28</td>
<td>2.38</td>
<td>-0.68</td>
<td>-14.69</td>
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<td>(7.49)</td>
<td>(6.60)</td>
<td>(6.14)</td>
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<td>$\gamma_{t+1}'$</td>
<td>-3.46</td>
<td>-0.41</td>
<td>-1.44</td>
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<td>(5.89)</td>
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<td>Risk$_t$</td>
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<td>(30.90)</td>
<td>(29.55)</td>
<td>(28.97)</td>
<td>(32.09)</td>
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<td>$Sur_{t+5}'$</td>
<td>-1.01</td>
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<td>(2.91)</td>
<td>(2.01)</td>
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<td>(1.44)</td>
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<td>Capital flows$_t$</td>
<td>4.59</td>
<td>-3.38</td>
<td>-3.40</td>
<td>-9.73</td>
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<tr>
<td></td>
<td>(7.41)</td>
<td>(4.52)</td>
<td>(3.66)</td>
<td>(4.26)</td>
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<td>R-squared</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.25</td>
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Dependent variable is the difference between the actual zero-coupon ten-year bond yield and the forecasted value from the Rudebusch-Wu (2004) model. Standard errors in parentheses. Bold = statistically significant at 5% level.
Table 3. RSW decompositions

<table>
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<td>Change in residual</td>
<td>-52.2</td>
<td>-52.2</td>
<td>-52.2</td>
<td>-52.2</td>
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<td>Due to $\pi_{t+10}$</td>
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<td>0.0</td>
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<td>0.0</td>
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<tr>
<td>Due to $y_{t+1}^e$</td>
<td>2.4</td>
<td>0.3</td>
<td>1.0</td>
<td>-4.7</td>
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<tr>
<td>Due to Risk$_t$</td>
<td>2.7</td>
<td>1.3</td>
<td>0.9</td>
<td>-1.7</td>
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<tr>
<td>Due to $Sur_{t+5}^e$</td>
<td>-1.2</td>
<td>-2.9</td>
<td>-3.9</td>
<td>-5.2</td>
</tr>
<tr>
<td>Due to Capital flows$_t$</td>
<td>-7.1</td>
<td>5.8</td>
<td>4.4</td>
<td>-6.8</td>
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Decomposition using change in each variable from 2004:1-2005:4 and coefficient estimates in Table 2.

Table 4. Alternative decomposition for RSW model

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<td>Difference in residual</td>
<td>-33.7</td>
<td>-33.7</td>
<td>-33.7</td>
<td>-33.7</td>
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<tr>
<td>Due to $\pi_{t+10}$</td>
<td>-3.0</td>
<td>-1.4</td>
<td>0.4</td>
<td>8.5</td>
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<td>Due to $y_{t+1}^e$</td>
<td>-2.9</td>
<td>-0.3</td>
<td>-1.2</td>
<td>5.7</td>
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<tr>
<td>Due to Risk$_t$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Due to $Sur_{t+5}^e$</td>
<td>0.8</td>
<td>1.9</td>
<td>2.6</td>
<td>3.4</td>
</tr>
<tr>
<td>Due to Capital flows$_t$</td>
<td>3.5</td>
<td>-4.3</td>
<td>-6.4</td>
<td>-33.4</td>
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<tr>
<td></td>
<td>Basis points</td>
<td>Percent of total</td>
<td>Basis points</td>
<td>Percent of total</td>
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<td>Actual-baseline</td>
<td>-125.2</td>
<td>-121.8</td>
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<td>Contribution of Budget surplus</td>
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<td>Interest rate risk</td>
<td>-16.3</td>
<td>13.0</td>
<td>-17.2</td>
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<td>GDP growth</td>
<td>7.2</td>
<td>-5.7</td>
<td>7.3</td>
<td>-6.0</td>
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<td>Capital flows rate</td>
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<td>9.1</td>
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<tr>
<td>Federal funds rate</td>
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<td>38.4</td>
<td>-46.6</td>
<td>38.2</td>
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<tr>
<td>Bond rate</td>
<td>-10.9</td>
<td>8.7</td>
<td>-20.6</td>
<td>16.9</td>
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Historical decompositions computed from six variable VAR with four lags estimated over 1984:1-2005:4. Variables are $Sur_{t+5}$, Risk, $y_{t+1}$, Capital flows, $f_t$, $\pi_{t+10}^e$, $i_{10,t}^e$. 

28
Figure 1. Ten year Treasury bond rate and federal funds rate, 1984:1 to 2005:4.

Yields are average of last month of quarter. Refer to the data appendix for further information and sources for all data used in this paper.
Figure 2. Ten-year-out expected inflation rate and actual CPI inflation, 1984:1-2005:4.

Expected inflation as described in Appendix. Actual inflation is 4-quarter CPI inflation rate.
Figure 3. One-year-ahead expected GDP growth and actual GDP growth, 1984:1-2005:4.

Expected GDP growth as described in Appendix. Actual growth is change in log of GDP in chained (2000) dollars over preceding four quarters.
Figure 4: U.S. federal budget surplus and 5-year CBO projection as a percent of GDP, 1984:1-2005:4.

CBO projection as described in Appendix. Actual is from FRB St. Louis.
Figure 5. Interest rate volatility measures, 1984:1-2005:4.

Trailing 36-month rolling standard deviation of monthly changes in the ten-year bond yield.
Figure 6. BEA data on capital flows as percent of GDP, 1984:1-2005:4.

Data as described in Appendix.
Figure 7. Official and private purchases of all U.S. bonds, four-quarter cumulative figures, 1984:1-2005:4.

Data as described in Appendix.
Figure 8. Official capital flows, BEA data and FRBNY data, 1984:1-2005:4

Data as described in Appendix.

Data supplied by Eric Swanson.
Figure 10. Response of real ten-year bond yield to one standard deviation shocks. Endogenous variables are $i_{t}^{10} \pi_{t+10}^{e}$, $f_{t}^{e}$, $\pi_{t+10}^{e}$, Capital flows $s_{t}^{e}$, $\gamma_{t+1}^{e}$, Risk $S_{t+5}^{e}$. 4 lags. Capital flows measured by FRB New York measure of official purchases of U.S. Treasury securities. 1984:1-2005:4. Dashed lines are two-standard error confidence bands.
Figure 11. Response of real ten-year bond yield to one standard deviation shocks. Endogenous variables are $i_{10,t} \pi^e_{t+10}, f_{t} \pi^e_{t+10}$, Capital flows, $y^e_{t+1}, \text{Risk}_t, Su^e_{t+5}$. 4 lags. Capital flows measured by BEA measure of official purchases of U.S. Treasury securities. 1984:1-2005:4. Dashed lines are two-standard error confidence bands.
Figure 12. Response of real ten-year bond yield to one standard deviation shocks. Endogenous variables are $i_{10,t}$, $\pi^{e}_{t+10}$, $f_{t-1}$, $\pi^{e}_{t+10}$, Capital flows, $y^{e}_{t+1}$, Risk, $Sur^{e}_{t+5}$. 4 lags. Capital flows measured by BEA measure of private and official purchases of U.S. Treasury securities. 1984:1-2005:4. Dashed lines are two-standard error confidence bands.
Figure 13. Response of real ten-year bond yield to one standard deviation shocks. Endogenous variables are $i_{10,t}$, $\pi^e_{t+10}$, $f_{t-1}$, $\pi^e_{t+10}$, Capital flows, $y_{t+1}^e$, Risk, $Sur^e_{t+5}$. 4 lags. Capital flows measured by BEA measure of private and official purchases of all U.S. bonds. 1984:1-2005:4. Dashed lines are two-standard error confidence bands.
Figure 14. Actual and baseline forecasts of real ten-year bond yield, by measure of capital flows used in VAR.

From 6-variable VAR described above.

From 6-variable VAR described above.

Shocks derived from 6-variable VAR described above.