

Graphing Calculators in Calculus

(Using a **TI-81** Calculator)

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Summary of Graphing Calculators in Calculus

You should be able to perform easily and efficiently all of the following tasks on your individual graphing calculator:

1. Do arithmetic calculations.
2. Define and evaluate functions.
3. Graph functions, and change the viewing window in meaningful ways.
4. Trace the graph of a function.
5. Find zeros of a function.
6. Find intersection points of the graphs of two functions.
7. Make a function table from a formula.
8. Find maxima and minima of a function.
9. Find the derivative of a function at a point, and graph derivative over an interval.
10. Find the definite integral of a function over an interval (LHS/RHS).
11. Graph the slope field of a differential equation, and sketch a solution curve.

#1: Using a TI-81 Graphing Calculator

Introduction

Graphing calculators and computer graphing software are indispensable tools in studying and doing mathematics. For this course you are **required** to have a graphing calculator available to you at all times during class, when doing your homework, and while taking exams. Although any calculator from the following list is acceptable, we **very highly recommend** that you use a calculator from the **Texas Instruments TI-83/84** series.

Texas Instruments TI-81, TI-82, **TI-83/84** series, TI-85, TI-86, or TI-89

Casio fx/cfx-7000/9000 series

Sharp EL-9000 series

Hewlett-Packard 48/49 series

Class demonstration, instruction, and discussion will all utilize a calculator from the **TI-83/84** series. Although the other listed calculators can perform most of the desired operations, those calculators might be more difficult to use on some operations that are expected in the course. Handouts are available for all of the **Texas Instruments** calculators that are listed.

The purpose of handout #1 is to guide you through learning how to use the basic calculator features that are not related directly to functions and graphing. You will type or key-press the items in **bold**.

Keyboard Layout

Take a few moments to become familiar with the layout of the keyboard. The lower central portion has the numerical digits. Arithmetic operation keys are on the lower right side. The **arrow** keys form a rectangle on the top right. Just above most keys are printed two additional labels:

- (1) A light blue label which you can access by first pressing and releasing the light blue **2nd** key on the top left of the keyboard, and
- (2) An gray letter or symbol label which you can access by first pressing and releasing the **ALPHA** key on the top left of the keyboard.

Note: If your screen is blank when you press the **ON** key in the lower left corner, press and release the **2nd** key once and then hold down the **up arrow** key to increase the screen contrast until you see something on the screen. Use the **2nd** and **down arrow** keys to reduce the contrast.

Arithmetic Operations

Practice the following calculations along with other similar calculations of your own design until you are comfortable and proficient with basic arithmetic calculations. After you type in each calculation to be done, press the **ENTER** key in the lower right corner of the keyboard. Use the **CLEAR**, **arrow**, and **DEL** (delete) keys to make typing corrections. Enter each of the calculations as one formula without breaking the formula down into simpler pieces.

$$5 + 7$$

$$3.6 - 8.25$$
 (Use the subtraction key on the right side, just above the addition key.)

$$4 \times 2$$
 (Note that multiplication is displayed on the screen by the "*" symbol.)

$$9 \div 4$$
 (Note that division is displayed on the screen by the "/" symbol.)

$$2 \wedge 3$$
 (The ^ key, just above the \div key, is for exponents: 2^3 .)

$$-4 + 9$$
 (Use the negation (-) key on bottom row for negatives, NOT the subtraction key.)

$$3 \times -6$$
 (Use the negation key, NOT the subtraction key.)

$$2 + 3 \times 4$$
 (Where are the implied parentheses?)

$$(2 + 3) \times 4$$
 (Parenthesis keys are above the 8 and 9 keys.)

$$2 \times 3 + 4$$
 (Again, where are the implied parentheses?)

$$2 + 3 \div 4$$
 (Implied parentheses?)

$$(2 + 3) \div 4$$
 (Parentheses must be used to calculate $\frac{2+3}{4}$.)

$$2 \div 3 \times 4$$
 (What would you enter to calculate $\frac{2}{3 \times 4}$?)

When in doubt about which operations are performed first, either try a simple similar example or use parentheses to clarify what you intend. What should you enter to calculate $\frac{4+7}{2+3}$?

$$5^2$$
 (Press the 5 key first and then the x^2 key on middle left side.)

$$(3 + 4)^2$$

$$5^{-1}$$
 (Press 5 and then x^{-1} key on middle left.)

$$\frac{1}{2.4} \quad \frac{1}{2+3}$$
 (Try each twice: first use the \div key, and then use the x^{-1} key.)

- $\sqrt{4}$ (Press light blue **2nd** key first, then x^2 key with light blue $\sqrt{\quad}$ label.)
- $\sqrt{12.25}$ (Parentheses are not necessary.)
- $\sqrt{9 + 16}$ (Parentheses are required.)
- $\sqrt{9} + \sqrt{16}$ (Is this result the same as the last result?)
- $\pi \div 2$ (To type π , press **2nd** key first and then \wedge key with light blue label " π ".)
- $2 \times \pi$
- 2π (Omit the \times key on this and the next three examples;
- $3 \sqrt{4}$ these four examples illustrate "implied" multiplication.)
- $2 (3 + 4)$
- $1 / 2 \pi$ (Is the result what you expected?)

Last Entry

Type again the calculation $2 (3 + 4)$ and press the **ENTER** key. Now press the blue **2nd** key and then the **ENTER** key with the light blue **ENTRY** label above it. Notice that your last formula entered reappears on the screen for you to make changes. Use the **arrow** keys to change the "3" to a "5" in order to compute $2 (5 + 4)$. Press the **ENTER** key as usual to carry out the calculation.

Edit your last entry (**2nd ENTRY**) again by first placing the flashing rectangular cursor over the digit "5" (use **left arrow** key). Press the **INS** key (next to the light blue **2nd** key). You are now in the "insert" mode instead of the "typeover" mode. Notice that the cursor is a flashing underline rather than a rectangle. Press the **3** key and notice what happens. Then press the **7** key. If you press an **arrow** key, the cursor goes back to the typeover mode as indicated by the flashing rectangular cursor. Press the **ENTER** key. (The cursor does not have to be at the end of the formula.)

Note: Since we've become aware of different cursor styles, watch what happens to the cursor style when you press the light blue **2nd** key. Press the **2nd** key again and watch. If you ever mistakenly press **2nd**, you can cancel by pressing **2nd** again. Look at the cursor to see whether **2nd** is activated. Similarly, watch the cursor style as you slowly press the **ALPHA** key a couple of times.

Variables

Type the calculation: $2 + 3$ ENTER. Notice that the answer is 5. Now press the \times key and the 4 key. The display screen reads "Ans*4". What is the result when you press ENTER?

"Ans" is a variable that stores your last calculated value (as opposed to your last entered formula). Now enter $50 - \text{Ans}$. (Press **2nd** key and then ANS key in lower right corner of keyboard.) The value of the variable Ans changes after each new formula calculation.

If you wish to calculate again the last formula entry without editing it, just press the ENTER key -- as many times as you wish to perform the calculation.

Example: Press 3 and then the ENTER key. The value of Ans is now 3.

Type $2 \times \text{Ans}$ ENTER. (Use the **2nd** and ANS keys.) The value of Ans is now 6.

Press the ENTER key repeatedly to see the value of Ans repeatedly doubled.

The result of each calculation is automatically stored in the variable Ans. Values can also be stored in single-letter variables by using the **STO** key located in the lower left corner of the keyboard. Carry out the following example entries and observe what happens:

5 STO	ENTER	(STO key flips cursor to ALPHA; just press key with label A.)
2 + A	ENTER	(Press the ALPHA key and then the key with label A. The value stored in variable A is used in the calculation.)
A \div 2 STO	ENTER	(The value of A/2 is displayed and also stored in variable B.)
A \times B	ENTER	(Current values of variables A and B are used in calculation.)
7 STO	ENTER	(The value of A is changed, but formula A/2 is not recalculated.)
B	ENTER	(Value of B did not change when a new value was stored in A.)

Experiment with other examples using variables to be sure you understand how they work in formulas and calculations.

You should practice repeatedly all the features discussed in this handout (and each later handout) until you are comfortable and proficient with them. You must be able to use these features easily and efficiently. These handouts show you the most important features you will need in calculus. Consult your calculator manual for further details and features.

#2: Functions and Graphing on a TI-81 Calculator

The purpose of handout #2 is to learn how to define, evaluate, and graph functions with your calculator. A TI-81 calculator can work simultaneously with up to four user-defined functions. Our first example will be the linear function $y = f(x) = x - 2$. Type or key-press the items in **bold**.

DEFINE the function by pressing the **Y=** key in the upper left corner and typing **X - 2** after the "**Y₁**=".

Use the key labeled "**X|T**" to type the independent variable. (Remember **CLEAR**, **DEL**, **INS**, and **arrow** keys to edit.) Press **QUIT** (light blue label on **CLEAR** key) to get back to the Home Screen.

Note 1: In function mode you must use the letter "**X**" as the name of the independent variable for any "**Y =**" function, no matter what the independent variable might be named in your actual problem. For example, if your problem uses $g(p) = p^2$, you must use $Y_1 = X^2$ on the calculator.

EVALUATE the function you have defined by using **Y₁** as the name of the dependent variable. To type the "**Y₁**" symbol, press the light blue (**2nd**) **Y-VARS** key first, and then press the **1** key for **Y₁**. Practice the examples, and observe the output. Remember that our example function is subtraction of 2 from the value of the independent variable **X**.

3 STO> X (We want to evaluate $f(3)$, that is, $f(x) = x - 2$ when $x = 3$)

Y₁ (The value of **Y₁** depends upon the current value of **X**, namely 3.)

6 STO> X (Change the current value of **X**.)

Y₁ (So the current value of **Y₁** changes correspondingly.)

4 STO> X

5 Y₁ + 3 (Notice the implied multiplication.)

Note 2: Alternately, a menu option may be selected by highlighting the option using the **arrow** keys and then pressing the **ENTER** key.

Note 3: The symbol "**Y₁**" is used as the name of the dependent variable (whose current value depends upon the current value of the independent variable "**X**").

GRAPH the function you have defined by pressing the **ZOOM** key on the top row, choosing option **8** for **Integer**, and pressing the **ENTER** key (maybe twice). The graph of your function is plotted in a viewing window that extends from -48 to 47 along the x -axis and from -32 to 31 along the y -axis. Tick marks are placed every 10 units along each axis. Press the **RANGE** key on the top row to see these specifications for the viewing window. The variables $Xmin$, $Xmax$ and $Ymin$, $Ymax$ describe the extent of each coordinate axis. The variables $Xscl$ and $Yscl$ describe the numerical distance between tick marks along these axes. The values of the window variables may be changed.

Press the **GRAPH** key on the top row to see the graph again. Use the **arrow** keys to move the *free-moving cursor* (+ sign) around the viewing window. Notice the x - and y -coordinates of the point at the center of the cursor. Use the free-moving cursor to write down the coordinates (with comma and parentheses) of three sample points of your choice from the graph of the function.

Press the **TRACE** key on the top row. You will see the X-shaped *trace cursor* on the graph of the function. Notice also the x - and y -coordinates of the graph point located at the cursor. Use the **right** and **left arrow** keys to move the trace cursor along the graph of the function. Notice in this example that the y -coordinate is always 2 less than the x -coordinate.

Note 4: The trace arrow keys restrict the possible x -coordinates of the points that can be specifically computed since the trace cursor moves in jumps from one screen pixel (tiny, square picture element) to another. In the **ZOOM Integer** window, the pixel steps are all 1 unit.

It is frequently more convenient to have the cursor move in steps of .1 instead of 1. Press the **RANGE** key and enter the following values:

$$\begin{array}{lll} Xmin = -4.8 & Xmax = 4.7 & Xscl = 1 \\ Ymin = -3.2 & Ymax = 3.1 & Yscl = 1. \end{array}$$

Then press the **GRAPH** key. Again move the free-moving cursor around to see the pixel steps. Finally, experiment with the trace cursor. This viewing window is called "ZDecimal" on the TI-83 calculator.

Note 5: More than one function can be defined and graphed at the same time. We will use $y = g(x) = 1 - x$ as our second example function.

Press **Y=** and **ENTER** to get down to the line " Y_2 =". Type **1 - X** and press **GRAPH**.

Press **TRACE**. Move trace cursor to the point where $x=.7$, and watch what happens when you press the **up** and **down arrow** keys.

#3: Changing the Viewing Window on a TI-81 Calculator

The purpose of handout #3 is to learn how to move the viewing window around the coordinate plane. Our two examples are the following exponential functions: $y = f(x) = 0.5(2^x)$ $y = g(x) = 2(0.4)^x$

SELECT functions for graphing by pressing the **Y=** key, **CLEAR**ing any previously defined functions, and entering $.5 \times 2^X$ for Y_1 and $2 \times .4^X$ for Y_2 . Since the Y_1 and Y_2 functions have highlighted equal signs, they are "selected" for graphing. Press the **RANGE** key and enter the "ZDecimal" viewing window values from handout #2. Press the **GRAPH** key, and notice the small box in the upper right corner of the screen indicating that the calculator is busy working.

Go back to **Y=**. Press the **left arrow** key to move the cursor over the first equal sign. Press the **ENTER** key once to "deselect" Y_1 . Press the **right arrow** key to see more clearly that the equal sign is no longer highlighted. Press **GRAPH** and observe only one function graph (which one?). Although the deselected Y_1 function is not plotted, it is still defined and can be used and evaluated from the Home Screen.

Press **Y=**, highlight the equal sign on Y_1 , press **ENTER**, and press **GRAPH**. While graphing you could press the **ON** key to stop graphing if desired. Observe the possible values of x and y when you use the **arrow** keys to move the free-moving cursor.

CHANGE the viewing **RANGE** to make x vary from -2 (negation key, not subtraction key) to 4 with tick marks every 1 unit and to make y vary from -2 to 10 with tick marks every 2 units. **GRAPH**. What is different about the values of x and y when you now use the **arrow** keys to move the free-moving cursor? The previous ZDecimal settings select values for the window variables so that the pixel steps are 0.1 in all directions. Most other choices for values of the window variables lead to more awkward sizes for the pixel steps. **TRACE** uses the pixel steps for x -coordinates and computes the function values for y -coordinates. Experiment with trying different values of the window variables and using **TRACE**.

ZOOMING is a shortcut to making certain kinds of changes in the values of the window variables. Press **RANGE** and enter the nice ZDecimal values for the window variables. We can also zoom in and out from whatever is our current viewing window. To set zooming factors press the **ZOOM** key and the **4** key to choose **SetFactors**. Change both the $XFact$ and $YFact$ variables to have the value **2** instead of 4.

ZOOM IN: Press the **ZOOM** key, press the **2** key to choose **Zoom In**, press the **ENTER** key to accept the origin as the zoom center, and when graphing is finished press the **CLEAR** key to stop further zooming. Use the **arrow** keys to move the free-moving cursor, and observe that the pixel steps of 0.05 are exactly half of the previous 0.1 pixel steps. Unfortunately, the origin itself is not exactly at a pixel point. Tick marks appear farther apart on the screen, but actually represent the same numerical distance as before the zoom. Press **RANGE** and notice that zooming has multiplied the previous min and max values by $\frac{1}{2}$ but has not changed the scale values for tick marks. Change the scale values to .5, and view the **GRAPH**.

ZOOM OUT: Press **ZOOM** and choose **Zoom Out**. Use the **arrow** keys to move the free-moving cursor to the point **(1.525, 1.025)** . Press the **ENTER** key to zoom out centered on this point. Observe how the axes are off-center. When the graphing is finished, press **ENTER** again to zoom out again on the same point. Finally, press **CLEAR** to stop further zooming. In **RANGE**, change both scale variables to 2, and press **GRAPH**.

ZOOM BOX: An alternate method of zooming in is to form a rectangular box to be the new viewing window. Let's apply this method to zoom in on the point of intersection of our two example function graphs. Choose **ZOOM Box**. Use the **arrow** keys to move the free-moving cursor to any corner of the new viewing rectangle desired. Press the **ENTER** key. Then use the **arrow** keys again to move the cursor to the opposite corner of that rectangle, and press **ENTER**. Use **ZOOM Box** to keep making new rectangle selections repeatedly until the x - and y -coordinates of the cursor do not change in, say, the fifth decimal place when the cursor is moved one pixel step in each direction. Write down (using comma and parentheses) the intersection point with coordinates rounded to four decimal places. *Answer:* (0.8614, 0.9084)

Note 1: None of the zooming commands affect the scale values, that is, the numerical distances between tick marks on the axes. After you have zoomed in and/or out to obtain the viewing window you desire, you may then want to press **RANGE** and enter suitable values for $Xscl$ and $Yscl$. You may also wish to "clean up" the values of $Xmin$, $Xmax$, $Ymin$, and $Ymax$. Press **GRAPH** to view the results.

Note 2: If you wish grid dots displayed within the viewing window, press the **MODE** key. Then use **arrow** keys to highlight **GridOn** with the flashing rectangle, and press **ENTER**. Finally, press **GRAPH** to see the result. **MODE** can also be used to turn off the grid dots and make other changes in how the graphing is done.

#4: Formula Tables and Data Plots on a TI-81 Calculator

The purpose of handout #4 is for you to learn how to use formula tables and data plots. Our example exponential function has formula $y = 0.5(2^x)$. You should press the **Y=** key and enter this formula for Y1 as before. The example data table is given to the right. We will enter this data into the calculator after we learn how to use a formula table.

x	y
-2	3.05
-1	2.55
0	1.94
1	1.64
2	1.25
3	1.07
4	0.84

FORMULA TABLES

The TI-82, TI-83/84, and TI-86 calculators have a powerful and flexible table feature. Some of that power can be mimicked on the TI-81 calculator by writing a program to do similar things. The following very simple program captures some of the table features, but it is not very flexible.

First we must enter the program. Press the **PRGM** key, highlight **EDIT**, and press the number of the first unused program slot. (Note that the cursor is already in the **ALPHA** style, so you should NOT press the **ALPHA** key when you type in the program name.) Type the name of the program (**TABLE**), and press the **ENTER** key. Type in the program itself using the key-press hints given in parentheses. Press the **ENTER** key at the end of each line. Remember to press the **ALPHA** key to type an alphabetic letter or gray symbol. The comma is on the period key. If you need to make corrections, use the **INS**, **DEL**, and **arrow** keys in the natural way.

Input X (Press **PRGM**, highlight **I/O**, and select **Input**.)
Input S (Press **STO>** to type "-".)
6 → Arow (Press **VARS**, highlight **DIM**, and select **Arow**.)
2 → Acol
1 → I
Lbl 1 (Press **PRGM**, and select **Lbl**.)
X → [A] (I , 1) (Use **2nd** key and **1** to type the light blue label [A].)
Y1 → [A] (I , 2) (Use **2nd Y-VARS**, and select Y1.)
X + S → X
IS>(I , 6) (Press **PRGM**, and select **IS>**(.))
Goto 1 (Press **PRGM**, and select **Goto**)
Disp [A] (Press **PRGM**, highlight **I/O**, and select **Disp**.)

When you have finished typing in the program, press the (**2nd**) **QUIT** key.

Before you execute the **TABLE** program, be sure you have entered a specific function for Y1. This table program uses ONLY the Y1 function. From the Home Screen, press the **PRGM** key, select the **TABLE** program, and press **ENTER**. At the first question mark, type a starting value of 0 for X , and press **ENTER**. At the second question mark, type a step size of .5, and press **ENTER**.

The first column of the table gives values for X , and the second column gives the corresponding values of Y1. Press the **ENTER** key (repeats the last command) to make another table starting at -2 and using .0001 for the step size. To display any hidden parts of the table on the Home Screen, press **2nd [A] ENTER**, and use the **left/right arrow** keys to scroll.

DATA PLOTS

Before making a data plot, deselect all of the functions defined by formulas under **Y=**. You may do this either by removing the highlights on all equal signs (handout #3) or by choosing **Y-VARS OFF All-Off** and pressing the **ENTER** key. (You may need to press **ENTER** again.)

Press the (**2nd**) **STAT** key, **arrow** to **DATA**, and select **Edit**. Type in (alternately) the values for x and y from the given data table. Press (**2nd**) **QUIT** when you are finished. For the given data points, enter a suitable set of **RANGE** values such as -5, 5, 1 for X and 0, 5, 1 for Y (*Min, Max, Scl*). Select **STAT DRAW Scatter**, and press the **ENTER** key. The data plot is easily erased, so you may have to plot it again.

MODEL THE DATA using a function defined by a formula. You can use trial and error to determine possible values for the parameters a and b in the family of exponential functions with formula $y = b \cdot a^x$ so that the graph of the function fits the data points as best as you can make it. The specific formula should be entered as one of the selected **Y=** functions. For example, start by entering the formula 2.1×0.7^X for the function Y3. Again select **STAT DRAW Scatter**, and press the **ENTER** key to see how well this specific model fits the given data. Modify the values of the parameters a and b to try to get a better fit.

#5: Left and Right Sums on a TI-81 Calculator

The purpose of handout #5 is to implement the computation of left- and right-hand sums on the calculator. Suppose we wish to estimate the area under the graph of $f(x)$ over the interval from $x=a$ to $x=b$ by forming the appropriate left-hand and right-hand sums for various values of n . For any choice of n , the increment in x would be $h = \frac{b-a}{n}$, and the equally spaced values of x would be

$$\left\{ \begin{array}{l} a = x_0 \\ x_1 = x_0 + h = a + h \\ x_2 = x_1 + h = a + 2h \\ x_3 = x_2 + h = a + 3h \\ \vdots \\ b = x_n = x_{n-1} + h = a + nh \end{array} \right.$$

Given $f(x)$, a , b , and a choice for n , the left- and right-hand sums, L and R , may be written:

$$L = \sum_{i=0}^{n-1} f(a+ih) \cdot h \quad \text{and} \quad R = \sum_{i=1}^n f(a+ih) \cdot h, \quad \text{where } h = \frac{b-a}{n}.$$

Notice that the only dissimilarity between the left-hand sum and the right-hand sum is in the lower and upper limits of summation. Finally, as the value of n becomes arbitrarily large, the values of L and R both approach the area under the graph of the function $f(x)$ over the interval $a \leq x \leq b$.

To implement the computation of left-hand sums on the calculator, we will write a program, that is, a list of the steps for the calculator to do. We need to type the program into the calculator only once. Then to compute a left-hand sum, we define the particular function and request the calculator to perform the program steps ("execute the program"). We can similarly implement the computation of right-hand sums. A brief *outline* of the program we will create later is as follows:

- a. Prompt for values of the parameters a , b , and n .
- b. Compute the increment, $h = \frac{b-a}{n}$.
- c. Starting with a sum of zero, repeatedly add each new product $f(a+ih) \cdot h$ to the previously accumulated sum.
- d. Display the final sum.

CREATE NEW PROGRAMS by typing program instructions into the calculator:

1. Press the **PRGM** key, highlight **EDIT**, and select the first unused **Prgm#**.
2. Type in the name of your program (**LHS**) following the "Prgm#:", and press the **ENTER** key. (Note that the flashing cursor is set into the **ALPHA** mode so you can just press the keys corresponding to the letters in the program name.)

- Type in the program itself using the key-press hints given in parentheses. Press the **ENTER** key at the end of each line. Remember to press the **ALPHA** key to type an alphabetic letter. The comma (,) key is on the period (.) key. If you need to make corrections, use the **INS**, **DEL**, and **arrow** keys in the natural way.

Input A (Press the **PRGM** key, highlight **I/O**, and select **Input**.)

Input B

Input N

(B - A) / N → H (Press the **STO>** key to type "-".)

0 → L (L will be the accumulated sum of products, initially set to 0.)

0 → I (This line would be **1 → I** in the RHS program.)

Lbl 1 (Press the **PRGM** key, highlight **CTL**, and select **Lbl**.)

A + I * H → X

L + Y1 * H → L (Use **2nd Y-VARS Y1** to type the Y1.)

IS>(I, N - 1) (Select **PRGM**, **CTL**, **IS>**(.) (Use **IS>(I, N)** in the RHS program.)

Goto 1 (Press the **PRGM** key, highlight **CTL**, and select **Goto**.)

Disp L (Press the **PRGM** key, highlight **I/O**, and select **Disp**.)

Stop (Press the **PRGM** key, highlight **CTL**, and select **Stop**.)

- When you have finished typing in the program, press the **2nd QUIT** key.
- To type in the program for computing right-hand sums, repeat the instructions in steps 1-4. Use **RHS** for the name, replace "L" by "R" everywhere, change **0→I** to **1→I**, and change the "IS>" statement to: **IS>(I, N)**
- If you need to modify a program after you have typed it into the calculator, press the **PRGM** key, highlight **EDIT**, and select the program you want to modify. Use the **INS**, **DEL**, and **arrow** keys in the natural way to help you make any changes. When you are finished, press **2nd QUIT** key.

EXECUTE PROGRAMS LHS and RHS to approximate, for example, the area under the graph of $f(x) = x^3$ over the interval from $x = 1$ to $x = 3$, using $n = 100$ subdivisions:

- Enter the formula x^3 as the Y1 function. Remember that LHS and RHS were written to use only the Y1 function.
- Press the **PRGM** key, highlight **EXEC**, and select the **LHS** program. Press the **ENTER** key to execute the program from the Home Screen.
- Enter the values **1**, **3**, and **100** (one value at each question mark) for the values of the parameters a , b , and n .
- Wait until the calculation is done. (Answer should be 19.7408.)
- Repeat steps 2-4 to compute the corresponding RHS. (Answer should be 20.2608.)
- Enter the formula $(L + R) / 2$ on the Home Screen to compute the average of the left- and right-hand sums. (Answer should be 20.0008.)

#6: Calculus Features on a TI-81 Calculator

In addition to the operations that we have previously learned (see earlier handouts), there are some powerful features on the TI-82, TI-83/84, TI-85, and TI-86 calculators that provide shortcuts to various calculus calculations. Unfortunately, the older TI-81 calculator does not have most of these features. This handout will review how to perform various calculus calculations on a TI-81 calculator. Consult your TI-81 calculator manual for further details of the features discussed below.

For this handout, define example functions $f(x) = x^3 - 3x^2 + 2.5$ $g(x) = x - 0.4$ and respectively. Graph both functions using ZDecimal settings. Type or key-press items in **bold**.

1. *Evaluate a function.* For example, compute $f(1.6183)$ and $g(1.6183)$. (Ans: -1.11853 and 1.2183)

1.6183 STO> X (Store 1.6183 as the current value of the independent variable X .)

Y₁ (Value of the dependent variable corresponding to current value of X .)

Y₂

Notes: **TRACE** gives function evaluation, but only at pixel-based values of X .
The **TABLE** program provides another way to evaluate functions.

2. *Find zeros of a function.* For example, the first of three zeros of $f(x)$ lies between -1 and 0 at about -0.5 for x . (Answer: $x = -0.8100379$)

Repeatedly use **ZOOM Box** or **ZOOM ZoomIn** to isolate a zero until the value of x from pixel to adjacent pixel does not change at the desired level of accuracy. You may wish to adjust differently the X and Y **ZOOM Factors** depending on the nature of the function. Use **TRACE** along with **ZOOM** to obtain more precise calculation of the y -coordinates on the graphs.

3. *Find intersection points of two graphs.* For example, the first of the three intersection points of $f(x)$ and $g(x)$ is at about -1 for x . (Ans: (-0.98738, -1.38738) is the intersection point.)

Repeatedly use **ZOOM Box** or **ZOOM ZoomIn** to isolate an intersection point until the value of x from pixel to adjacent pixel does not change at the desired level of accuracy. You may wish to adjust differently the X and Y **ZOOM Factors** depending on the nature of the functions. Use **TRACE** along with **ZOOM** to obtain more precise calculation of the y -coordinates on the graphs. Use the **up** and **down arrow** keys to see whether the function value (y -coordinate) at the intersection point does not change at the desired level of accuracy between the two functions.

4. Find local maxima and minima of a function on an interval. For example, find local maxima and local minima of the function $f(x)$ on the interval $[1,3]$. (Ans: $(2, -1.5)$ is local minimum point.)

Again, the repeated use of **ZOOM Box** and **ZOOM Zoom In**, along with **TRACE**, will allow you to isolate the coordinates of local maximum and minimum points. When you **Zoom In** more rapidly on the y -coordinate, the maximum or minimum point appears sharper and more clearly distinguished.

5. Find and graph the derivative of a function. For example, compute $f'(2.5)$ and graph $f'(x)$.

Home Screen: **2.5 STO> X**

First store the desired value of x .

MATH NDeriv(Y1, .001)

See 3.750001 as approximate value of $f'(2.5)$

Note: NDeriv uses a central difference quotient with 0.001 as this value of h .

Y= Screen: Define $Y_3 = \text{MATH NDeriv}(Y_1, .00001)$ and deselect the Y_2 function.

Reset the ZDecimal RANGE values, and **GRAPH** the functions Y_1 and Y_3 .

Y_3 can be used as $f'(x)$ for computation and graphing. Now deselect Y_3 in **Y=**.

6. Find the definite integral of a function over an interval. For example, compute $\int_{0.2}^{2.0} f(x) dx$

There is no separate definite integral calculator on the TI-81. You can use the **LHS** and **RHS** programs (and the average of **L** and **R**) to approximate the value of a definite integral. Using $N = 100$, you should get $L = 0.54262116$, $R = 0.47263716$, and 0.5076916 as the average. You can also use the Fundamental Theorem of Calculus to compute the exact value as 0.5076.

7. Draw on the graphing screen.

Use (**2nd**) **DRAW ClrDraw** **ENTER** to clear any previous drawings. Deselect Y_2 in **Y=**.

DRAW Draw F Y1 - 2 will provide a temporary graph of $f(x) - 2$. (**ClrDraw** when done.)

8. Draw tangent lines to graph of a function. For example, draw the tangent line to the graph of $f(x)$ at $x = 2.2$. Be sure that only $f(x)$ is selected on the **Y=** screen.

Home Screen: **2.2 STO> X**

(Set the value of x at the point of tangency.)

Y1

(See that $y = -1.372$ at the point of tangency.)

NDeriv(Y1, .00001) (Slope is $m = 1.32$ at the point of tangency.)

Draw F -1.372 + 1.32 (X - 2.2) (Use **2nd DRAW**. Tangent line formula.)

Y= Screen: $Y_4 = -1.372 + 1.32 (X - 2.2)$ Reset ZDecimal RANGE values, and **GRAPH**.

#7: Differential Equations on a TI-81 Calculator

The purpose of handout #7 is to implement graphing the slope field of a differential equation and sketching an approximate solution curve using Euler's method. The programs for graphing a slope field and for using Euler's method must be typed into your TI-81 calculator by hand. Type or key-press items in **bold**.

EXECUTE THE PROGRAMS

0. First, be sure that you have type the programs into your TI-81 calculator as described below.
1. In the window $-4 \leq x \leq 4$ and $-3 \leq y \leq 3$, let's use the example differential equation $\frac{dy}{dx} = x + y$
2. Press **Y=** and enter the expression **X + Y** for the Y_1 . Press **RANGE** and enter the appropriate values for the x and y intervals, using a scale value of 1 in each direction.
3. Press the **PRGM** key, highlight **EXEC**, and select the **SLOPEFLD** program. Press the **ENTER** key to execute the program from the Home Screen.
4. Watch the slope field being graphed. When finished, press **2nd QUIT**.
5. Press the **PRGM** key, highlight **EXEC**, and select the **EULER** program. Press the **ENTER** key to execute the program from the Home Screen.
6. Type **1** and **ENTER** at the first prompt to keep the plotted slope field in the viewing window. Enter **-1** for the initial value of x and **.2** for the initial value of y . Enter **2** to graph a solution curve in both directions from the initial point. When finished, press **2nd QUIT**.
7. Press the **ENTER** key to repeat the Euler program, type **0** at the first prompt to clear the viewing window, and use the same initial point to make a graph in both directions. **2nd QUIT**.
8. Without clearing the viewing window, repeat the Euler program several times using different initial points and choices for the direction(s) to graph from the initial point.

TYPE IN THE PROGRAMS

1. The slope field program is adapted from a program by Mark Howell that appears on page 113 of *Technology Resource Manual for Calculus* by Finney, Thomas, Demana, and Waits.
2. See handout #5 for general instructions on creating new programs on the TI-82.
3. Some of the new key-press combinations are: For **All-Off**, use **2nd Y-VARS Off**. For **ClrDraw**, use **2nd DRAW**. For **DispGraph** and **Input**, use **PRGM I/O**. For **Lbl**, **Goto**, **If**, **While**, use **PRGM CTL**. For **<**, **>**, and **=**, use **2nd TEST**. For **Xmin**, **Xmax**, **Ymin**, **Ymax**, and **ΔX** , use **VARS RNG**. For **Line**, use **2nd DRAW**. For **abs**, use **2nd ABS**.

SLOPE FIELD PROGRAM (SLOPEFLD)

```

14 - L
18 - W
(Ymax - Ymin) / L -> V
(Xmax - Xmin) / W -> H
All-Off
ClrDraw
DispGraph
0 - R
Ymin + V / 2 -> Y
Lbl 1
R + 1 -> R
0 - C
Xmin + H / 2 -> X
Lbl 2
C + 1 -> C
Y1 - M
-M * H / 2 + Y -> S
M * H / 2 + Y -> T
X - H / 2 -> P
X + H / 2 -> Q
If abs ( T - S ) > V
Goto 3
Lbl 4
Line ( P , S , Q , T )
X + H -> X
If C < W
Goto 2
Y + V -> Y
If R < L
Goto 1
Stop
Lbl 3
Y + V / 2 -> T
Y - V / 2 -> S
( T - Y ) / M + X -> Q
( S - Y ) / M + X -> P
Goto 4

```

EULER'S METHOD PROGRAM (EULER)

```

Disp "TO CLEAR WINDOW "
Disp "ENTER 0 ."
Input C
If C = 0
ClrDraw
All-Off
(Xmax - Xmin) / 95 -> D
Disp "INITIAL X ."
Input P
Disp "INITIAL Y ."
Input S
Disp "1 LEFT , 2 BOTH , "
Disp "3 RIGHT ."
Input C
If C < 2
Goto 2
P -> X
S -> Y
Lbl 1
If X ≥ Xmax
Goto 2
X + D -> Q
Y + Y1 * D -> T
Line ( X , Y , Q , T )
Q -> X
T -> Y
Goto 1
Lbl 2
If C > 2
Goto 4
P -> X
S -> Y
Lbl 3
If X ≤ Xmin
Goto 4
X - D -> Q
Y - Y1 * D -> T
Line ( X , Y , Q , T )
Q -> X
T -> Y
Goto 3
Lbl 4
DispGraph
Stop

```