Problem of the Week Contest

Spring 2019, Number 6

Department of Mathematics, Gettysburg College

Posted: Friday, March 22.

Solutions Due: Friday, March 29, 5:00 p.m..

Send solutions to bkenney@gettysburg.edu or put solutions in the marked envelope in the hallway outside Glatfelter 215.

The contest is open to all Gettysburg College students. Up to three people may work together on a submission. Make sure your name is on your submission and that any sources are properly cited!

Powerful numbers

Let \( m \) be an integer (that is, a whole number — perhaps positive, perhaps negative, perhaps zero). Let us say that \( m \) is a SPA number if there are positive whole numbers \( k \) and \( j \) such that

\[
m = \frac{m^k + m^j}{2}.
\]

QUESTION. Find all the SPA numbers. In your solution, explain how you know your list is complete.