

Posted: Monday, November 11, 2019

Deadline: **11:59 PM on Sunday, November 17, 2019**

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Rules (please read):

- The contest is open to everyone.
- Individuals or teams of at most three members may submit solutions.
- Do not discuss the problem with anyone other than members of your team.
- You may use any source, written or electronic, but all sources must be properly cited.
- You may use any computational tools.
- Your solution will be graded on a 0–4 point scale.
- All your assertions must be completely and fully justified. At the same time, you should aim to be as concise as possible; avoid overly lengthy arguments and unnecessary components. Your grade will be based on both mathematical accuracy and clarity of presentation.
- Either send your solution to [bbajnok@gettysburg.edu](mailto:bbajnok@gettysburg.edu) or put it in Béla's departmental mailbox by the above deadline.
- Consistently successful participants will receive the *Paul Mugabi Mathematics Problem Solving Award*.

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### **A million tokens**

Consider a sequence of tokens numbered  $1, 2, 3, \dots, 1000000$ . Each token has one side colored orange and the other side colored blue. Originally, each token has its orange side facing upward. At time  $t = 1$ , you turn each token upside down (and thus each token will have its blue side facing upward). Then, at time  $t = 2$ , you turn every second token upside down (and therefore, tokens numbered 2, 4, 6, etc. will have their orange side facing up). At time  $t = 3$ , you turn every third token, and so on, until time  $t = 1000000$ , when you turn every 1000000th (that is, the last) token upside down. How many tokens will have their orange side facing up at the end?